tainties to which the experiment is subject. However, these results are considered highly encouraging and indicate that the further development of the method is a worthy pursuit.

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Optimum Toroidal Pressure Vessel Filament Wound along Geodesic Lines

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Nomenclature

x,y	= Cartesian coordinates
y',y''	= first and second derivative of y with respect to x
σ	= principal stress, lb/in.
ρ	= principal radius of curvature
R	= radial distance of any point on toroid to axis of rotational symmetry
ω	= angle $\tan^{-1}(dy/dx)$
β	= variable wrap angle, i.e., angle of the filament with the meridian at any point
v	= quantity $(1 + y'^2)^{1/2}/y'$ or $1/\sin\omega$
n	= factor of safety
N	= total number of turns of one set of filaments required for construction of toroid
p	= gage pressure of inflation gas
T	= strength of filament
S_b	= stress in direction of either set of filaments at a point, lb/in.
A,C	= constant quantities
$f(\beta),F(\beta)$	= functions of wrap angle

Subscripts

i	= toroid inner equator
o	= toroid outer equator
m	= toroid parallel circle passing through origin of coordinates
1	= meridional direction
2	= circumferential direction

Discussion

LET $a - a$ be the axis of rotational symmetry of a toroidal surface, the meridian of which is shown only partly in Fig. 1a by the curve OM. In Fig. 1b are shown two filaments passing through a point M of the toroid; these filaments are arranged symmetrically with respect to the meridian at that point.

Considering as a free body the circular band generated by the arc OM rotating 360° about the axis of rotational symmetry and writing equilibrium of forces along this axis gives

$$2\pi(R_m + x)\sigma_1 \sin\omega = \pi[(R_m + x)^2 - R_m^2]p \qquad (1)$$

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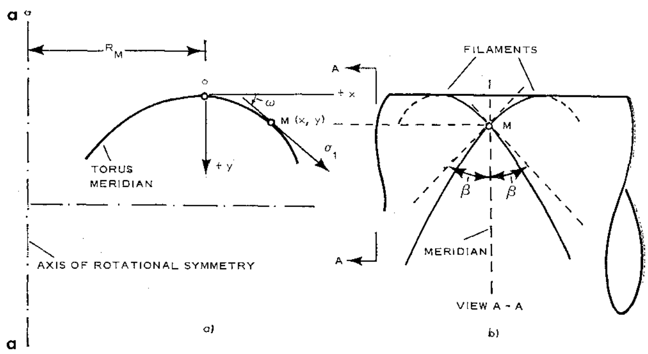


Fig. 1 a) Toroid meridional section; b) coordinates and wrap angle.

Solving Eq. (1) for σ_1 yields

$$\sigma_1 = \frac{x(2R_m + x)p}{2(R_m + x)\sin\omega} \qquad (2)$$

Substituting Eq. (2) into

$$(\sigma_1/\rho_1) + (\sigma_2/\rho_2) = p \qquad (3)$$

(see Ref. 1, p. 160), noting that

$$\rho_1 = (1 + y'^2)^{3/2}/y'' \qquad (4)$$

$$\rho_2 = (R_m + x)/\sin\omega \qquad (5)$$

and solving the resulting equation for σ_2 yields

$$\sigma_2 = \frac{R_m + x}{\sin\omega}\left[1 - \frac{x(2R_m + x)y''}{2(R_m + x)(1 + y'^2)^{3/2}\sin\omega}\right] \qquad (6)$$

Noting that

$$\sin\omega = \frac{\tan\omega}{(1 + \tan^2\omega)^{1/2}} = \frac{y'}{(1 + y'^2)^{1/2}} \qquad (7)$$

Eqs. (2) and (6) become, respectively,

$$\sigma_1 = \frac{x(2R_m + x)(1 + y'^2)^{1/2}p}{2(R_m + x)y'} \qquad (8)$$

$$\sigma_2 = \frac{(R_m + x)(1 + y'^2)^{1/2}}{y'}\left[1 - \frac{x(2R_m + x)y''}{2(R_m + x)y'(1 + y'^2)}\right]p \qquad (9)$$

Let β be the wrap angle, i.e., the variable angle between the meridional line at a point and each of the two filaments passing through this point (see Fig. 1b), N be the total number of turns of the one set of filaments required for the construction of the toroid, and T be the strength of the filaments in pounds. The number of filaments which cross the unit length of the parallel circle passing through the point M is $N/2\pi(R_m + x)$; and the strength S_b in pounds per inch of each set of filaments is

$$S_b = NT/2\pi(R_m + x)\cos\beta \qquad (10)$$

Let P be the force carried by the $N/2\pi(R_m + x)$ filaments of one set in the direction of the filaments. From Fig. 2 it is clear that

$$P = S_b \cos\beta \qquad (11)$$

The component of P along the meridional direction is $P\cos\beta$ or $S_b\cos^2\beta$. Hence, for the two sets of filaments this component is $2S_b\cos^2\beta$. If n is the desired factor of safety, the force $2S_b\cos^2\beta$ corresponds to an actually applied stress $2S_b\cos^2\beta/n$, which acts along the meridional direction; therefore it is equal to σ_1. Hence,

$$n\sigma_1 = 2S_b\cos^2\beta \qquad (12)$$

Similarly, the component of P along the parallel circle is

$S_b \cos\beta \sin\beta$; for the two sets of filaments this component is $2S_b \cos\beta \sin\beta$. This force, which is normal to the meridional direction, acts over a length equal to $\cot\beta$; therefore, per unit length, this force is $2S_b \sin\beta \cos\beta/\cot\beta$ or $2S_b \sin^2\beta$, which is equal to $n\sigma_2$. Hence,

$$n\sigma_2 = 2S_b \sin^2\beta \tag{13}$$

Substituting Eq. (10) into Eqs. (12) and (13), letting

$$NT/\pi n = Ap \tag{14}$$

and solving the resulting equations for σ_1 and σ_2 yields

$$\sigma_1 = \frac{Ap}{R_m + x}\cos\beta \qquad \sigma_2 = \frac{Ap}{R_m + x}\frac{\sin^2\beta}{\cos\beta} \tag{15}$$

Eliminating the parameter angle β between Eqs. (15) results in

$$\sigma_1^2 + \sigma_1\sigma_2 = A^2p^2/(R_m + x)^2 \tag{16}$$

Substituting Eqs. (8) and (9) into Eq. (16) yields

$$\frac{x(2R_m + x)(1 + y'^2)}{2y'^2}\left[\frac{x(2R_m + x)}{2(R_m + x)} + R_m + x - \frac{x(2R_m + x)}{2}\frac{y''}{y'(1 + y'^2)}\right] = \frac{A^2}{R_m + x} \tag{17}$$

Equation 17 is the differential equation of the meridional section of the optimum filament-wound torus. To solve this equation let $v = (1 + y'^2)^{1/2}/y'$. The resulting equation is a linear differential equation of the dependent variable v^2. Its solution is

$$v^2 = \frac{4A^2(R_m + x)^2 - C}{[x(R_m + x)(2R_m + x)]^2} \tag{18}$$

where C is the constant of integration.

Noting that $(1 + y'^2)^{1/2}/y' = v = 1/\sin\omega$, equating right-hand sides of Eq. (8) and the first of Eqs. (15), and solving the resulting equation for $\cos\beta$ yields

$$\cos\beta = [x(2R_m + x)/2A]v \tag{19}$$

Eliminating v between Eqs. (18) and (19) and simplifying results in

$$\sin\beta = C^{1/2}/2A(R_m + x) \tag{20}$$

Let β_m be the wrap angle at $x = 0$; then Eq. (20) gives

$$\sin\beta_m = C^{1/2}/2AR_m \tag{21}$$

Combining Eqs. (20) and (21) yields

$$(R_m + x)\sin\beta = R_m \sin\beta_m \tag{22}$$

This equation states that the sine of the wrap angle at any point is inversely proportional to the distance from the axis of rotational symmetry. According to this remarkable property, the filaments, which at each point constitute a set

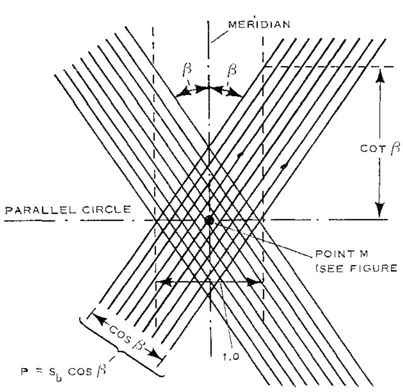


Fig. 2 Arrangement of two sets of filaments at a point.

balancing under the existing stress condition [Eqs. (12) and (13)], follow paths of geodesic lines on the surface of the toroid (see Ref. 2, p. 96). This means that under inflation pressure there will be no sliding of the filaments, because, as they are located on the surface of the toroid, the filaments meet at an angle compatible with the stress condition everywhere, and, at the same time, they follow paths (geodesic lines) which are compatible with pure tension, which is the only load that flexible filaments can carry efficiently.

Noting that $C = 4A^2R_m^2 \sin^2\beta_m$ (from Eq. 21) and $x + R_m = R_m \sin\beta_m/\sin\beta$ [from Eq. (22)], from which $x = R_m (\sin\beta_m/\sin\beta - 1)$ and $x + 2R_m = (\sin\beta_m/\sin\beta + 1)R_m$, and substituting into Eq. (18) and simplifying results in

$$v = \pm\frac{2A \sin^2\beta \cos\beta}{R_m^2(\sin^2\beta_m - \sin^2\beta)} \tag{23}$$

Let R_i and R_o be the radii of the inner- and outer-equatorial line of the torus, x_i and x_o the corresponding values of the coordinate x, and β_i, β_o the corresponding values of the wrap angle β. Noting that

$$v = 1/\sin\omega = (1 + y'^2)^{1/2}/y' \tag{24}$$

and that $\sin\omega = 1$ at $x = x_o$, and $\sin\omega = -1$ at $x = x_i$, Eq. (23) gives, respectively,

$$\frac{2A \sin^2\beta_o \cos\beta_o}{R_m^2(\sin^2\beta_m - \sin^2\beta_o)} = +1$$
$$\frac{2A \sin^2\beta_i \cos\beta_i}{R_m^2(\sin^2\beta_m - \sin^2\beta_i)} = -1 \tag{25}$$

Substituting equations

$$x_o = R_o - R_m \text{ (positive)} \qquad x_i = R_i - R_m \text{ (negative)} \tag{26}$$

into Eq. (22) yields

$$R_o \sin\beta_o = R_m \sin\beta_m = R_i \sin\beta_i \tag{27}$$

Substituting Eq. (24) into (23) and solving the resulting equation for y' gives

$$y' = \pm 1 \div \left[\frac{4A^2 \sin^4\beta \cos^2\beta}{R_m^4(\sin^2\beta_m - \sin^2\beta)^2} - 1\right]^{1/2} \tag{28}$$

where the plus sign corresponds to $x > 0$ or $\beta < \beta_m$ and the minus sign corresponds to $x < 0$ or $\beta > \beta_m$ [see Eq. (22)].

Solving the first of Eqs. (25) for $2A/R_m^2$ and substituting the resulting equation into Eq. (28) yields

$$y' = \frac{dy}{dx} = \pm 1 \div \left[\left(\frac{\sin^2\beta_m - \sin^2\beta_o}{\sin^2\beta_m - \sin^2\beta}\frac{\sin^2\beta}{\sin^2\beta_o}\frac{\cos\beta}{\cos\beta_o}\right)^2 - 1\right]^{1/2} \tag{29}$$

which upon integration gives

$$y = \pm\int_0^x \frac{dx}{\left[\left(\frac{\sin^2\beta_m - \sin^2\beta_o}{\sin^2\beta_m - \sin^2\beta}\frac{\sin^2\beta}{\sin^2\beta_o}\frac{\cos\beta}{\cos\beta_o}\right)^2 - 1\right]^{1/2}} \tag{30}$$

Solving Eq. (22) for x/R_m yields

$$x/R_m = (\sin\beta_m/\sin\beta) - 1 \tag{31}$$

Differentiate Eq. (31):

$$dx = -(R_m \sin\beta_m/\sin^2\beta)\cos\beta d\beta \tag{32}$$

Substituting Eq. (32) into Eq. (30) and noting that $\beta = \beta_m$ at $x = 0$ results in

$$\frac{y}{R_m} = \mp\int_{\beta_m}^{\beta} \frac{(\sin\beta_m \cos\beta/\sin^2\beta)d\beta}{\left[\left(\frac{\sin^2\beta_m - \sin^2\beta_o}{\sin^2\beta_m - \sin^2\beta}\frac{\sin^2\beta}{\sin^2\beta_o}\frac{\cos\beta}{\cos\beta_o}\right)^2 - 1\right]^{1/2}} \tag{33}$$

Table 1 Determination of the wrap angle of a toroid with $R_i/R_o = 0.65$

β		ω		ω,					
Deg	Min	Deg	Min	rad	$f(\beta)$	$\Delta\omega$	$f(\beta)_{av}$	$f(\beta)_{av}\Delta\omega$	$\Sigma f(\beta)\Delta\omega$
42	00	−90	00	−1.5708	0.3693				0
						0.2621	0.3653	0.0957	
41	34	−74	59	−1.3087	0.3613				0.0957
						0.2609	0.3481	0.0908	
40	20	−60	02	−1.0478	0.3349				0.1865
						0.2633	0.3195	0.0841	
38	27	−44	57	−0.7845	0.2840				0.2706
						0.2609	0.2454	0.0640	
36	13	−30	00	−0.5236	0.2068				0.3346
						0.2617	0.1576	0.0412	
33	54	−15	00½	−0.2619	0.1083				0.3758
						0.2619	0.0542	0.0142	
31	19′17″	0	0	0	0				0.3900
						0.2604	−0.0526	0.0137	
29	53	+14	55	+0.2604	−0.1051				0.4037
						0.2626	−0.1522	0.0400	
28	22	+29	58	+0.5230	−0.1993				0.4437
						0.2614	−0.2375	0.0621	
27	13	+44	57	+0.7844	−0.2757				0.5058
						0.2669	−0.3043	0.0812	
26	24	+60	14½	+1.0513	−0.3328				0.5870
						0.2608	−0.3512	0.0916	
25	56	+75	10½	+1.3121	−0.3695				0.6786
						0.2587	−0.3736	0.1004	
25	47	+90	00	+1.5708	−0.3777				0.7790

$$I_1 = \int_{\omega=-90°}^{\omega=0} f(\beta)d\omega \cong 0.3900 \qquad I_2 = \int_{\omega=0}^{\omega=90°} f(\beta)d\omega \cong 0.7790 - 0.3900 = 0.3890$$

$$I_1 - I_2 = +0.0010 \cong 0$$

(The minus sign is taken when $x > 0$ or $\beta < \beta_m$, and the plus sign is taken when $x < 0$ or $\beta > \beta_m$.) For brevity, let

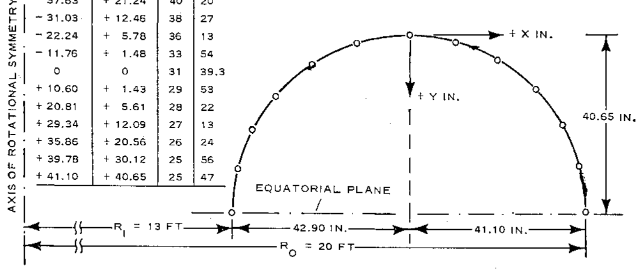
$$F(\beta) = \frac{(\cos\beta/\sin^2\beta)}{\left[\left(\dfrac{\sin^2\beta_m - \sin^2\beta_o}{\sin^2\beta_m - \sin^2\beta}\,\dfrac{\sin^2\beta}{\sin^2\beta_o}\,\dfrac{\cos\beta}{\cos\beta_o}\right)^2 - 1\right]^{1/2}} \tag{34}$$

It is clear that the y values at $\beta = \beta_o$ and $\beta = \beta_i$, which are the maximum y values, must be equal. The points corresponding to these y values lie in the equatorial plane of the toroid. This means that

$$-\int_{\beta_m}^{\beta_o} F(\beta)d\beta = \int_{\beta_m}^{\beta_i} F(\beta)d\beta \tag{35}$$

For a given ratio R_i/R_o, there is one angle β_m and only one from which the geometry of the meridian of the torus can be determined. Once the problem has been solved for several values of R_i/R_o, a graph can be plotted of β_m against R_i/R_o which can be used to determine the wrap angle and geometry of the meridian for any torus configuration.

The problem, because of its complexity, can be solved on a trial and error basis, either by longhand calculations or preferably by digital computers.



X	Y	ANGLE β DEG	MIN
−42.90	+40.65	42	00
−41.57	+30.72	41	34
−37.63	+21.24	40	20
−31.03	+12.46	38	27
−22.24	+5.78	36	13
−11.76	+1.48	33	54
0	0	31	39.3
+10.60	+1.43	29	53
+20.81	+5.61	28	22
+29.34	+12.09	27	13
+35.86	+20.56	26	24
+39.78	+30.12	25	56
+41.10	+40.65	25	47

Fig. 3 Meridional half section of filament-wound toroid with $R_i/R_o = 0.65$; coordinates and wrap angle.

It is clear that in the limiting case $R_i/R_o \rightarrow 1$, $R_o - R_i$ = const, the toroid degenerates to a cylinder, and the answer is well known. [Circular cylinder, wrap angle $\beta = \cot^{-1}(2^{1/2}) = 35°16'$ = const, geodesic lines are helices on the cylinder.]

The following steps show the procedure for the solution of the problem for a given ratio R_i/R_o ($R_i/R_o \leqq 1.0$):

1) Assume a value for the angle β_i.†
2) Determine β_o from Eq. (27).
3) Determine β_m from Eq. (36):

$$\cos 2\beta_m = \cos 2\beta_i + \frac{\sin\beta_i \sin 2\beta_i[1 - 2\sin^2\beta_o - \cos 2\beta_i]}{\sin\beta_i \sin 2\beta_i + 2\sin^2\beta_o(1 - \sin^2\beta_o)^{1/2}} \tag{36}‡$$

which can be derived by summing up Eqs. (25), solving the resulting equation for $\sin^2\beta_m$, and using the identity $\cos 2\beta_m = 1 - 2\sin^2\beta_m$.

4) Determine the angle ω for various values of the angle β ($\beta_o \leqq \beta \leqq \beta_i$) from Eq. (37):

$$\sin\omega = \frac{\sin^2\beta_m - \sin^2\beta}{\sin^2\beta_m - \sin^2\beta_o}\,\frac{\sin^2\beta_o}{\sin^2\beta}\,\frac{\cos\beta_o}{\cos\beta} = \frac{\cos\beta_o - \cos 3\beta_o}{\cos 2\beta_o - \cos 2\beta_m}\,\frac{\cos 2\beta - \cos 2\beta_m}{\cos\beta - \cos 3\beta} \tag{37}$$

which can be derived by combining Eqs. (23), (24), and the first of Eqs. (25).

5) Determine the function $f(\beta)$ from Eq. (38):

$$f(\beta) = \frac{\csc\beta}{\tan^2\beta + [2(1 - \cos 2\beta_m)/(\cos 2\beta_m - \cos 2\beta)]} \tag{38}$$

† A good β_i value for a first trial can be taken from the equation $\tan\beta_i = 2(2^{1/2}) \div [3(R_i/R_o) + 1]$, which was derived on the supposition that the (unknown) meridional section of the torus is a circle.

‡ Equation (36) was left unsimplified intentionally to contain the sine of the angle β_o as was found in step 2, in order to facilitate hand calculations and reduce the possibility of errors if other functions of the angle β_o were introduced.

6) See if the following equation is satisfied:

$$\int_{\omega=-90°}^{\omega=0} f(\beta)d\omega = \int_{\omega=0}^{\omega=90°} f(\beta)d\omega \qquad (39)§$$

If Eq. (39) is not satisfied, repeat the procedure for a different β_i value.

For the correct value of β_i, the nondimensional coordinates x/R_m and y/R_m of the meridian of the toroid can be found from Eqs. (40):

$$x/R_m = (\sin\beta_m/\sin\beta) - 1$$

$$\frac{y}{R_m} = \mp\sin\beta_m \int_0^{\omega} f(\beta)d\omega \qquad (40)$$

plus sign for $x > 0$ or $\beta < \beta_m$
minus sign for $x < 0$ or $\beta > \beta_m$

If R_i and R_o are given numerically, the radius R_m can be determined from Eq. (27), and the coordinates x and y can be calculated dimensionally from Eqs. (40).

In calculating values of ω (see step 4) for various values of the angle β, it is preferable to get equally spaced values of ω rather than of β, in order to get approximately equally spaced points on the meridional section of the toroid; this also facilitates the integrations shown on both sides of Eq. (39) since Simpson's rule can be used most effectively. Provision for equally spaced values of ω, however, requires two or three trials per value because β cannot be expressed as an explicit function of ω.

Numerical Example: $R_i = 13$ ft, $R_o = 20$ ft

The preceding method was applied to a toroid whose radii on the equatorial plane are 13 and 20 ft. The solution is given in a tabular form, which includes only the last step of approximation. As shown at the bottom of Table 1, Eq. (39) is satisfied only approximately because $I_1 - I_2 = 0.3900 - 0.3890 = 0.0010$ instead of zero. The accuracy, however, is considered satisfactory. Coordinates x and y were calculated from Eqs. (40), and corresponding values along with β values are given in a table in Fig. 3, where half the meridional section of the toroid also is plotted.

The most remarkable conclusion is that the meridional section of the toroid is very nearly circular, and for all practical purposes it can be taken as such. This conclusion, however, must not be generalized before other meridional sections are plotted for more R_i/R_o values. It can be seen easily that a circular torus does not lend itself to a filament-winding process because the stress ratio at points around a meridian varies in a way incompatible with the rate of change of the wrap angle. The numerical example, however, shows that a very nearly circular torus ($R_i/R_o = 0.65$) can be filament wound if the filaments start at a wrap angle $\beta_i = 42°$ on the inner equator. An explanation for this lies on the sensitivity of the circumferential stresses in a torus. In fact, if general equations for the principal stresses are written for a toroid of elliptical meridional section, it can be seen that it does not take much eccentricity in order for the circumferential stress pattern to change considerably.[3]

§ This is an equivalent equation of the condition $y_{(x=x_i)} = y_{(x=x_o)}$, which is given by Eq. (35). In fact, substituting Eq. (37) into Eq. (34) and simplifying results in

$$F(\beta) = \cos\beta \tan\omega/\sin^2\beta \qquad (i)$$

Differentiating Eq. (37) and simplifying yields

$$\cos\omega d\omega = \frac{\sin^2\beta_m \sin^2\beta - 2\sin^2\beta_m\cos^2\beta - \sin^4\beta}{\sin^3\beta\cos^2\beta} d\beta \qquad (ii)$$

Dividing Eqs. (37) and (ii) yields

$$\cot\omega\, d\omega = \frac{\sin^2\beta_m \sin^2\beta - 2\sin^2\beta_m\cos^2\beta - \sin^4\beta}{\sin\beta\cos\beta(\sin^2\beta_m - \sin^2\beta)} d\beta \qquad (iii)$$

Solving Eq. (iii) for $d\beta$ and multiplying the resulting equation and Eq. (i) gives

$$F(\beta)d\beta = \frac{\cot^2\beta}{\sin\beta}\,\frac{1-(\sin\beta_m/\sin\beta)^2}{1+(\sin\beta_m/\sin\beta)^2(2\cot^2\beta - 1)} d\omega$$

which reduces to $F(\beta)d\beta = f(\beta)d\omega$ if Eq. (38) is taken into account. From the last equation, the equivalence between Eqs. (35) and (39) is obvious.

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Flow Separation in Overexpanded Contoured Nozzles

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Reference 1 presented some data on separation due to overexpansion in contoured nozzles. Unfortunately, the experimental data are presented in a system of coordinates which bears little relation to the parameters describing the separation phenomenon, and, therefore, the significance of the results is masked This defect is mentioned by the authors of Ref. 1.

In Refs. 2–5, it has been shown that the significant parameters describing flow separation in overexpanded nozzles are the separation pressure ratio, i.e., the ratio of the wall pressure at the separation point to ambient pressure, and the separation Mach number. It also has been shown that all available experimental data on two-dimensional and conical nozzles of half angles up to 30° correlate adequately when plotted as a function of the Mach number in the separation plane. Reference 4 introduces a simple theory to account for the variation of the separation pressure ratio as a function of the Mach number and the specific heat ratio.

In attempting to relate contoured nozzle data to those of straight-walled nozzles, the following essential differences should be borne in mind:

1) *There are differences in the pressure gradient along and normal to the nozzle wall.* Information available to date indicates that variations in the longitudinal positive pressure gradient have a negligible effect on the separation pressure rise. The effect of a normal pressure gradient has not been investigated. It is to be expected that a normal gradient will either delay or cause earlier separation, depending on the sense of the gradient in the vicinity of the separation point.

2) *Because of the curvature of the nozzle wall, the geometry of the separated region generally differs from that encountered in straight-walled nozzles.* This difference may account for variations in the pressure rise occurring in the separated region.

Figure 1 presents experimental data for the ratio of pressure at the separation point to ambient pressure for contoured nozzles. The data are taken from Refs. 1, 6, and 7. The correlation for straight-walled nozzles of Ref. 4 is indicated also. The considerable scatter is probably due to the range of pressure gradients prior to separation and different separated region geometries that are encountered in contoured

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